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481. Proposed by ROGER E. MOORE, University of Wisconsin.

Show that if the coefficients in the binomial expansion of $(a - b)^n$ (n being a positive integer) be multiplied each to each by the corresponding terms of an arithmetical progression of $(n + 1)$ terms, then the algebraic sum of the $(n + 1)$ products will be zero.

SOLUTION BY WILLIAM R. RANSOM, Tufts College.

The $(k + 1)$st term of the progression is $(f + kd)$, and the $(k + 1)$st coefficient is $\dfrac{(-1)^k\underline{|n}}{\underline{|k}\,\underline{|n-k}}$. Then the sum in question is

$$\sum_{k=0}^{k=n}\frac{(-1)^k\underline{|n}}{\underline{|k}\,\underline{|n-k}}\cdot(f+kd).$$

We may insert at pleasure suitable powers of $a = b = 1$, and separate this sum into the two sums

$$f\sum_{k=0}^{k=n}\frac{(-1)^k\underline{|n}}{\underline{|k}\,\underline{|n-k}}a^kb^{n-k} - nd\sum_{k=1}^{k=n}\frac{(-1)^{k-1}\underline{|n-1}}{\underline{|k-1}\,\underline{|n-k}}a^{k-1}b^{n-k},$$

which equals $f(a - b)^n - nd(a - b)^{n-1}$ and so is evidently zero.

Also solved by N. P. PANDYA, HORACE OLSON, O. J. RAMLER, LOUIS O'SHAUGHNESSEY, R. M. MATHEWS, E. H. WORTHINGTON, and the PROPOSER.

GEOMETRY.

512. Proposed by J. L. RILEY, Stephenville, Texas.

Determine, geometrically, where the circle of curvature at any point of an ellipse again meets the ellipse.

SOLUTION BY NATHAN ALTSHILLER, University of Oklahoma.

Let (O) be the circle of curvature at a point P of the *conic* (C), t the tangent to (C) at P, and Q the other point common to (O) and (C). The two curves determine a pencil of conics, the lines t and PQ forming a degenerate conic of the pencil.[1] The pencil of conics determines an involution of points on the line at infinity (Desargues' theorem), the double elements E, F of which are conjugate with respect to every conic of the pencil.

Since the circle (O) belongs to the pencil, the circular points at infinity I, I' are corresponding elements in this involution of points. From the point P this involution of points is projected by pairs of lines in involution, and to the positive isotropic line PI corresponds the negative isotropic line PI'; hence, the involution is equilateral, *i. e.*, the double rays e, f (projecting from P the double points E, F) are orthogonal and bisect the angles formed by any pair of rays of the involution, in particular, the angles formed by t and PQ.

If the conic (C) has a center C at a finite distance, the lines a, b joining C to the two points E, F, conjugate with respect to (C) and lying on the polar of C with respect to (C), are themselves conjugate lines with respect to (C), and being parallel to e, f, are perpendicular to each other; hence, a, b are the axes of (C). The double elements e, f are, therefore, the parallels through P to the axes of (C). If (C) is a parabola, one of the double elements, say E, of the involution of points on the line at infinity is the point of contact of this line with the parabola; hence, the double element e of the involution of lines at P is parallel to the axis of the parabola.

These considerations suggest the following simple construction of the required point Q, *applicable to any conic:* Through the given point P draw a line parallel to an axis of the given conic and also a line PQ such that the parallel shall bisect the angle between PQ and the tangent at P to the conic. The line PQ will meet the conic again in the required point Q.

The point Q will coincide with P, if the tangent t is perpendicular to the line drawn through P parallel to the axis, and only in this case, *i. e.*, if P is a vertex of the conic. We thus find the known proposition that the vertices of a conic are the points, and the only points, at which the conic has a four-point contact with the osculating circle.

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[1] See, for instance, Veblen and Young, *Projective Geometry*, Vol. I, p. 134.